Master Maths
First year

2024-2025
Kasdi Merbah University

Homework 5

Ex. 1. Show that if σ is a polynomial of degree $n \geq 0$, then it is not discriminatory.
Hint : Take $\mu = L_{n+1}(x)dx$ where L_{n+1} is the Legendre polynomial of degree $n+1$ and dx is the Lebesgue mesure.

Ex. 2. Let $f(x) = x$, show that

$$E_{n,H}(f) = \frac{1}{2(n+1)}$$

Ex. 3. Let H the be Heaviside function, $f \in C^1[0,1]$. We have

$$E_{n,H}(f) \leq \frac{1}{2(n+1)}\|f'\|.$$

This approximation order and the coefficient are the best possible.

Ex. 4. Let H the be Heaviside function, $f \in C^1[0,1]$. We have

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Ex. 5. Let BV denote the class of continuous functions of bounded variation on $[0,1]$, and let $V(f)$ denote the total variation of $f \in BV$. Let σ be a bounded sigmoidal function. Show that if $f \in BV$, then

$$E_{n,\sigma}(f) \leq \frac{\|\sigma\|}{n+1}V(f)$$

where $\|\sigma\| = \sup_{x\in\mathbb{R}} |\sigma(x)|$