TP 1
Master Maths

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Université Kasdi Merbah

Error indicator and mesh adaptation in FreeFem++

1 A Classical Schema of mesh adaption

Algorithm 1

1. $i = 0$;
2. Let $\mathcal{T}_h^i$ an initial mesh.
3. loop
4. compute u_h^i the solution on mesh $\mathcal{T}_h^i$.
5. evaluate the level of error err.
6. if $err < \varepsilon$ break
7. compute the new local mesh size h_{i+1}.
8. construct a mesh according to prescribe the mesh size.

2 How to code using Freefem++

We need to define some new macros that allow us :

1. The computation of mesh size $h_{\max}$ of a given mesh Th.
2. The computation of the local indicator η_T in each triangle T of Th.
3. The construction of a new mesh according to prescribe mesh size.

2.1 Basic commands

1. **mesh** Th : Type for meshes.
2. **hTriangle** : gives the size of the current triangle T.
3. **lenEdge** : gives the length of the current edge e.
4. **average(g)** : the average of edge length.
5. **jump (g)** : the jump of the function g across edge.
6. **intalledges(Th)** :
7. **N** : gives the outward unit normal vector, N.x and N.y are its x and y components.
8. **adaptmesh** :

Exercise 1 *Write a Freefem++ code computs the mesh size.* $\Omega =]-100, 100[\times]-100, 100[$. *Observe that* $h_T = \frac{\int_T h_T}{|T|}$.

```
macro  MeshSizecomputation1(Th,Ph,h)
{
Ph area, lent;
varf varea(u,v)= int2d(Th)(hTriangle*v);
varf  vlent(u,v)= int2d(Th)(v);
area[] = varea(0,Ph); lent[]  = vlent(0,Ph);
...
}
```

```
macro  MeshSizecomputation2(Th,Ph,h)
{
fespace Vh(Th,P1) ;
Vh nbEperV, meshsize ;
varf vmeshsizen(u,v)=intalledges(Th,qfnbpE=1)(v) ;
varf vedgecount(u,v)=intalledges(Th,qfnbpE=1)(v/lenEdge) ;
...
}
```

Exercise 2 *Write a Freefem++ code computs the indicator* η_T *(for* $u = \tanh(xy/20)$

```
func u = tanh(x*y/20.);
func uxx= -(1./200)*tanh(x*y/20.)*(1-tanh(x*y/20.)^2)*y^2;
func uyy= -(1./200)*tanh(x*y/20.)*(1-tanh(x*y/20.)^2)*x^2;
func f = -uxx-uyy;
Vh uh=u;
varf vindicator(uu, chiT) = intalledges(Th)(chiT*lenEdge*square(jump(N.x*dx(uh)+N.y*dy(uh))))
                             +int1d(Th)(chiT*lenEdge*square(N.x*dx(uh)+N.y*dy(uh)))
                               +int2d(Th)(chiT*square(hTriangle*(f-dxx(uh)-dyy(uh))) );
fespace Ph(Th,P0) ;                                  //    a space of function constant by element
Ph rho, etaT ;
...
```

Exercise 3 *Write a Freefem++ code constructs of a new mesh according to prescribe mesh size.*

```
macro  ReMeshIndicator(Th,Ph,Vh,vindicator,coef)
{
Vh h=0;
MeshSizecomputation(Th,Vh,h);
Ph etaT;
etaT[ ]=vindicator(0,Ph);
```

```
etaT[ ]=sqrt(etaT[ ]);
real etastar= coef*(etaT[ ].sum/etaT[ ].n);
Vh fn,sigma;
varf veta(unused,v)=int2d(Th)(etaT*v);
varf vun(unused,v)=int2d(Th)(1*v);
fn[ ]  = veta(0,Vh);
sigma[ ]= vun(0,Vh);
fn[ ]= fn[ ]./ sigma[ ];
fn =  max(min(fn/etastar,3.),0.3333) ;
h = h / fn ;
        Th=adaptmesh(Th,IsMetric=1,h,splitpbedge=1,nbvx=10000);
}                                                    //  end of macro Remesh.
```

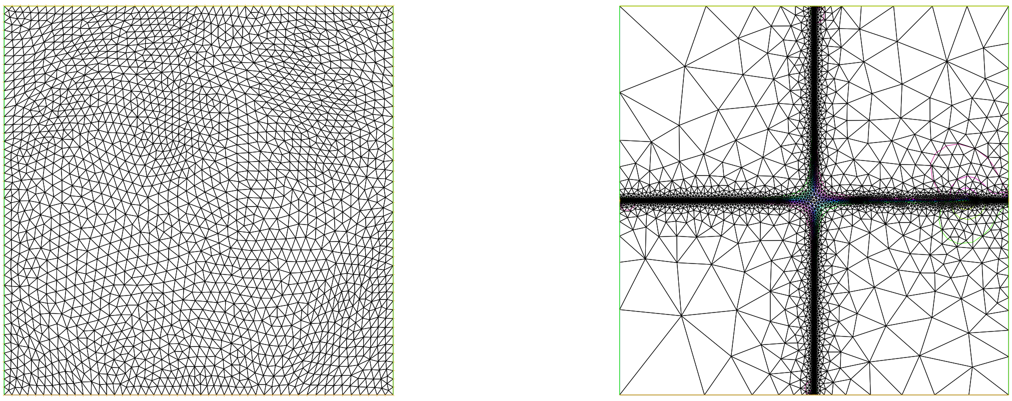
3 First PDE's Example : Laplace Equation

We consider Laplace's equation

$$\begin{cases} \text{Find } u : \Omega \to \mathbb{R} \text{ such that} \\ -\Delta u = f \ \text{ on } \Omega \\ u = g \text{ on } \partial\Omega. \end{cases} \tag{1}$$

where,

$$\Omega =]-100, 100[^2, \quad g = u = \tanh(xy/20), \quad f = -u_{xx} - u_{yy}$$



(a) Initial mesh (b) Adapted mesh

FIGURE 1 – The result that should you obtained after 10 iterations

Exercise 4 *The same quation for*

$$u(x,y) = \tanh(-100(y - 0.5 - 0.25\sin(2 * pi * x))) + \tanh(100(y - x)), \quad \Omega =]0,1[^2$$

4 Anisotropic Laplacian

4.1 Example

We consider the anisotropic Laplacian :

$$\begin{cases} \text{Find } u : \Omega \to \mathbb{R} \text{ such that} \\ -\varepsilon_1 \partial_1^2 u - \varepsilon_2 \partial_2^2 u = f \text{ in } \Omega \\ u = g \text{ on } \partial\Omega. \end{cases} \tag{2}$$

where,

$$\Omega =]-1,1[^2, \quad g = e^{-\pi x/\sqrt{\varepsilon}} \sin(\pi y), \quad f = 4(1-\varepsilon_1 x^2 - \varepsilon_2 y^2) e^{(-\varepsilon_1 x^2 - \varepsilon_2 y^2)}, \quad \varepsilon_1 = 0.05, \varepsilon_2 = 0.001$$

4.2 Example

We consider the anisotropic Laplacian :

$$\begin{cases} \text{Find } u : \Omega \to \mathbb{R} \text{ such that} \\ -\text{div } (A \nabla u) = 0 \text{ in } \Omega \\ u = g \text{ on } \partial\Omega. \end{cases} \tag{3}$$

where,

$$\Omega =]0,1[^2, \quad g = e^{-\pi x/\sqrt{\varepsilon}} \sin(\pi y), \quad A = \begin{pmatrix} \varepsilon & 0 \\ 0 & 1 \end{pmatrix}, \quad \varepsilon = 10^{-k}, k = 1,2,3$$

the exact solution is

$$u(x,y) = e^{-\pi x/\sqrt{\varepsilon}} \sin(\pi y)$$